Identifying inferiority by a stepdown procedure with feedback

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1 Introduction

Consider a balanced one-way layout where $\bar{X}_i \sim N(\mu_i, \sigma^2/n)$, $1 \leq i \leq k$, are independent estimates of a set of treatment means μ_i, and $S^2 \sim \sigma^2\chi^2_\nu/\nu$ is an independent estimate of the variance for some degrees of freedom ν. In this paper, any treatment satisfying $\mu_i = \max_{1\leq j\leq k} \mu_j$ is considered to be a best treatment, and the best treatment is not necessarily unique.

In many cases a comparison of the treatment means is most usefully done by comparing each treatment with the unknown best treatment or treatments within the group. This is an important goal if the experiment has been performed as a screening study to identify which treatments are worthy of further study, and also in situations where it is useful to know how inferior a particular treatment could be relative to the best treatment or treatments in the group. This goal complements other approaches to the comparisons of the treatment means, such as comparing all treatments with one particular treatment (which may be a placebo, say), or making all pairwise comparisons among the treatment means, both of which have been extensively studied in the statistical literature.

The subset selection approach to the problem of comparing each treatment with the unknown best treatment operates by constructing a subset of the treatments with a specific characteristic, and the pioneering work in this field was done by Gupta(1956). However, Gupta's method cannot reach the specific goal addressed in this paper which identifies all of the best treatments when there are several. From the perspective of multiple comparisons, the goal can be addressed through the construction of a set of simultaneous confidence intervals for the difference between each treatment and the best treatment (or the best of all the other treatments). The related works can see (Edwards and Hsu(1983)), Hsu(1984), Hsu(1996), and Tukey(1953). Finally, Hayter(2007) provides a sharper decision procedure that shares the characteristics of both subset selection procedures and multiple comparison procedures. It also has another advantage that provides upper bounds on how inferior the treatments are compared with the best treatments.

In this paper the case $k = 3$ is considered, and a stepdown procedure is proposed for identifying as many treatments as possible to be strictly inferior to the best treatment or treatments. This stepdown procedure has the novelty that it incorporates feedback from the first stage to the second. The fact that the required error properties of the procedure can be maintained with the implementation of feedback is an interesting result of this research, and it has wider implications to other standard stepwise procedures. Whereas the steps of a stepwise procedure are usually constructed to involve statistics that are unrelated to the statistics used at the previous steps, this new approach illustrates that improvements can be made by incorporating feedback from previous stages.

2 The new stepdown procedure with feedback

The new stepdown decision procedure with feedback operates as follows.
Step I:

- If $\frac{\bar{X}_{(3)}-\bar{X}_{(1)}}{S/\sqrt{n}} \leq d^{\alpha}_{3,\nu} = q^{\alpha}_{3,\nu}$, then the set W is empty. Stop.
- Otherwise, the treatment T_i corresponding to $\bar{X}_{(1)}$ is placed into the set W, and the procedure continues to Step II.

Step II: Calculate $d_{2,\nu}^{\alpha} = \sqrt{2}\ t_{\nu}^{\alpha/2}\left[1 - e^{a-b\left(\frac{\bar{X}_{(3)}-\bar{X}_{(1)}}{S/\sqrt{n}}\right)}\right]$.

- If $\frac{\bar{X}_{(3)}-\bar{X}_{(2)}}{S/\sqrt{n}} \leq d_{2,\nu}^{\alpha}$, then no further treatment is added to W. Stop.
- Otherwise, the treatment T_j corresponding to $\bar{X}_{(2)}$ is added to the set W. Stop.

The question now arises of whether there are any choices of a and b that allow this stepdown procedure to guarantee the specified error probability α throughout the parameter space. In order to investigate this essential question, this paper also establishes two properties of these error probabilities.

3 Contributions and conclusions

It turns out that there are many feasible choices for a and b. We can then define optimal values which provides the most improvement over the standard stepdown procedure. It is known that this new stepdown procedure with feedback will always be at least as good as the standard stepdown procedure with a constant critical point $d_{2,\nu}^{\alpha} = \sqrt{2}t_{\nu}^{\alpha/2}$ for all data values, since the sets W will either be identical or will contain two treatments for the new procedure and only one treatment for the standard procedure. The amount of improvement of the new procedure is largest for data values where $\frac{\bar{X}_{(3)}-\bar{X}_{(1)}}{S/\sqrt{n}}$ is just slightly greater than $d_{3,\nu}^{\alpha} = q_{3,\nu}^{\alpha}$, since this is the case where step II of the procedure is implemented and the critical point $d_{2,\nu}^{\alpha}$ of the new procedure has the greatest reduction compared with $\sqrt{2}t_{\nu}^{\alpha/2}$. Besides, the presented procedure with feedback is better than the other procedures for this problem discussed in section 1 (with the possible exception of the (Edwards and Hsu(1983)) procedure). Last, the new procedure with feedback allows equivalent sensitivity to the standard procedure but requires smaller sample size.

References

Broström, G. (1981). On sequentially rejective subset selection procedures. *Communications in Statistics - Theory and Methods*, A 10, 203-221.

Edwards, D. G. and Hsu, J. C. (1983). Multiple comparisons with the best treatment. *Journal of the American Statistical Association*, 78, 965-971.

Finner, H. and Giani, G.(1994). Closed subset selection procedures for selecting good populations. *Journal of Statistical Planning and Inference*, 38, 179-200.

Finner, H., Giani, G. and Strassburger, K. (2006). Partitioning principle and selection of good treatments. *Journal of Statistical Planning and Inference*, 136, 2053-2069.

Gupta, S. S. (1956). On a decision rule for a problem in ranking means. Ph.D. thesis, Institute of Statistics, University of North Carolina, Chapel Hill.

Hayter, A. J. (2007). A combination multiple comparisons and subset selection procedure to identify treatments strictly inferior to the best. *Journal of Statistical Planning and Inference*, 137, 2115-2126.

Hsu, J. C. (1984). Ranking and selection and multiple comparisons with the best. Thomas J. Santner and Ajit C. Tamhane (eds), *Design of Experiments: Ranking and Selection*, Marcel Dekker, New York.

Hsu, J. C. (1996). *Multiple comparisons, theory and methods*, Chapman & Hall, London.

Lam, K. (1986). A new procedure for selecting good populations. *Biometrika*, 73, 201-206.

Lin, Chen-ju and Hayter, A. J. (2008). A stepdown procedure with feedback for identifying inferiority among three treatments. To appear in the Biometrical Journal.

Tukey, J. W. (1953). The problem of multiple comparisons. *The Collected Works of John W. Tukey Vol. VIII. Multiple Comparisons: 1948-1983*, 1-300, Chapman & Hall, New York.