Systematic Approach for Portmanteau Tests in View of Whittle Likelihood Ratio

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1. Introduction

In time series model building, it is usual to verify the adequacy of a fitted model by computing residual autocorrelations. For this Box and Pierce (1970) proposed a test statistic

$$T_{BP} = n \sum_{k=1}^{m} \hat{r}_k^2, \tag{1·1}$$

where $\hat{r}_k$ is the sample autocorrelation of lag k of the estimated residual process. Here n is the sample size, and T_{BP} is called the portmanteau test statistic. Under the null hypothesis that the ARMA(p,q) model is adequate, Box and Pierce (1970) suggested that the distribution of T_{BP} is approximated by χ^2_{m-p-q}, "if m and n are moderately large". However, Davies et al. (1977) claimed that the χ^2_{m-p-q} -approximation is not adequate, i.e., showed that, even for moderately large n and $m = 20$, the true significance levels are likely to be much lower than predicted by asymptotic theory. Ljung and Box (1978) proposed an improved version of T_{BP}:

$$T_{LB} = n(n+2) \sum_{k=1}^{m} (n-k)^{-1} \hat{r}_k{}^2, \tag{1·2}$$

which is called the Ljung-Box test statistic. However, Ansley and Newbold (1979) reported that the asymptotic significance levels by T_{LB} yield a serious understatement. Various modified versions of portmanteau test can be found in e.g., Arranz (2005).

In many application fields, portmanteau tests, especially, T_{BP} and T_{LB}, have been widely used. It is very important to develop the systematic asymptotic theory which grasps the portmanteau tests from unified view. This paper elucidates that the portmanteau tests are essentially equivalent to a special form of Whittle likelihood ratio T_{PW} for the spectral density $f_{(\theta_1,\theta_2)}(\lambda)$ of (**??**) in Section 2, which tests whether the residual correlation parameter θ_2 satisfies $H : \theta_2 = 0$ or $A : \theta_2 \neq 0$. Then, it is shown that, under H, for any finite $m = dim\ \theta_2$, $T_{PW} \nrightarrow \chi^2_{m-p-q}$ in distribution as $n \to \infty$. This result is caused by the fact that T_{PW} uses the Whittle estimator $\hat{\theta}_1$ for the model $f_{(\theta_1,0)(\lambda)}$ and that $\hat{\theta}_2(\hat{\theta}_1)$ for the estimated model $f_{(\hat{\theta}_1,\theta_2)}(\lambda)$. As an auxiliary result we show that, if the time series structure has Bloomfield's exponential spectral model, then, for any finite m, $T_{PW} \to \chi^2_{m-dim\ \theta_1}$, in distribution under H.

Next we propose a natural Whittle likelihood ratio test T_{WLR} which is based on $\hat{\theta}_1$ and $(\tilde{\theta}_1, \tilde{\theta}_2)$ which is the Whittle estimator for the model $f_{(\theta_1,\theta_2)}(\lambda)$. Then it is shown (i) $T_{WLR} \to \chi^2_m$ in distribution under H, and (ii) $T_{WLR} \to$ a noncentral χ^2-distribution in distribution under a sequence of contiguous alternatives $A_n : \theta_2 = h/\sqrt{n}$. Numerical studies for (i) and (ii) are provided. They illuminate an interesting feature of T_{WLR}. Since the portmanteau tests are important benchmark statistics, our systematic studies for them give a unified view.

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