What happens in estimating the cointegrated VAR/VARMA models?

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The paper investigates what happens in estimating the cointegrated VAR/VARMA models. In estimating the model concerned, it does not check whether the stationarity and the invertibility conditions are satisfied. By means of Monte Carlo simulation, this paper shows some unpleasant situations even in simple data generated processes (DGPs).

Since Granger (1981) and Engle and Granger (1987), cointegrated models have been investigated by many literature; Engle and Granger (1987) for a single equation approach, Johansen (1988, 1991) for a system estimation approach. Also Yap and Reinsel (1995) (the Gaussian (conditional) likelihood approach), Lütkepohl and Claessen (1997), Poskitt (2003) (the echelon form combined with the error correction form), Takimoto and Hosoya (2004, 2006) (the Whittle likelihood approach) deal with ARMA cointegrated models. For finite sample performances of the cointegrating rank test, many authors have conducted simulation studies intensively for last twenty years. In many literature, especially, DGPs of Gonzalo (1994), Toda (1994, 1995) and Yap and Reinsel (1995) are utilized very often to investigate finite sample performances of the cointegrating rank test. Also Hubrich (2001) overviews the literature on systems cointegration tests and conducts some new simulations for VAR models. In estimating the coefficients of the model, the preceding papers does not check whether the set of estimated coefficients satisfies the root condition, which is assumed to be valid usually, but unrestricted optimization algorithm may not keep the root condition satisfied. This paper investigates what happens in estimating the cointegrated VARMA models by our root-modification procedure.

We summarize our Whittle-likelihood based estimation procedure, which consists of
(1) eigenvalue contraction method and/or spectral matrix factorization method
(2) penalty-imposed likelihood function maximization.
Those devices guarantee initial coefficient values and coefficient estimates to satisfy the root condition.

Our three-step algorithm for maximization is organized as this: (1) Step 1 produces a consistent initial estimate of β and residuals; (2) Step 2 estimates the ARMA coefficients

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for each pair of ARMA order (a, b) by substituting the disturbances with the residuals produced in Step 1. If the root condition of the estimated model are violated, the estimated set of ARMA coefficients is modified so as to be located inside the admissible set by the root-contraction mechanism or the canonical factorization mechanism of the MA spectrum; (3) For each pair of lag orders, Step 3 sets the estimated procedures in Step 2 as the initial values for maximizing iteration. To maximize the Whittle likelihood, our method uses an optimality algorithm in which penalty functions are added to the log likelihood in order to keep the ARMA coefficient estimates satisfying the root condition. We identify the ARMA order in the final stage by means of the *BIC*, producing the final selection of the final selection of the pair $(\hat{a}, \hat{b})$.

We conduct some simulations based on the trivariate AR(2) and ARMA(1,1) models which are examined by Yap and Reinsel (1995). Suppose that the null hypothesis of cointegration rank zero is given by $H_0 : rank(\Pi) = 0$, whereas the alternative hypothesis of rank three is given by $H_3 : rank(\Pi) = 3$. To test based on a direct computation of the LR statistics, it needs to estimate the full-rank stationary model under the null hypothesis. We show all the roots of $\det C(z) = 0$ and $\det B(z) =$ for the full-rank stationary model, and a set of estimated coefficients obtained by an unrestricted estimation procedure is not admissible. Also, we observe that the invertibility condition is violated and the MA representation by this set of coefficient is not proper in terms of the prediction error. By our canonical factorization mechanism, we can obtain a proper MA representation. Our simulation results indicate that a set of coefficients based on an unrestricted estimation procedure may not be admissible and our estimation procedure guarantees the root condition.

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