確率的回帰モデルの推測について

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以下の線形回帰モデル

$$y_i = \alpha + \boldsymbol{x}_i'\boldsymbol{\beta} + \epsilon_i \qquad i = 1, \ldots, T.$$

を考える。　ここで以下の仮定をおく。

Assumptions：

- k 個の説明変数 $\boldsymbol{x}_i$ は、確率ベクトル
- $(\boldsymbol{x}_i', \epsilon_i)'$ は、i.i.d. で４次の有界なモーメントを持つ
- $E[\epsilon_i] = 0, E[\boldsymbol{x}_i] = \boldsymbol{\mu}_x$, $V[\epsilon_i]$, $V[\boldsymbol{x}_i]$.
- ϵ_i は、$\boldsymbol{x}_i$ と無相関、($E[y_i|\boldsymbol{x}_i] = \alpha + \boldsymbol{x}_i'\boldsymbol{\beta}$ のとき成立する)

この仮定の下での回帰モデルの統計的推測についての私の最近の研究について紹介する。

モデルの推測は、説明変数 $\boldsymbol{x}_i$ が与えられたときの条件付き推測 (conditional inference) と $\boldsymbol{x}_i$ が与えられていないときの無条件な推測 (unconditional inference) に分けられる。

無条件な推測の場合：elliptical 分布の４次のモーメントについての特性、LSE の漸近共分散行列の特性、無相関だが独立でない説明変数と誤差項の場合の LSE の漸近共分散行列についてのシュミュレーション

- The effects of nonnormality on the market model in the class of elliptical distributions, Columbia=JAFEE international conference proceedings, (2000)
- Tail-thickness in terms of $cov(Xj^2, Xp^2)$ in the class of elliptical distributions, Journal of statistical planning and inference,(2004)
- The asymptotic covariance matrix of the least squares estimator in the stochastic linear regression model:the case of elliptically symmetric distribution, submitted (2007)
- The robustness of asset pricing models: coskewness and cokurtosis (with M. Ando), Finance Research Letters, (2006)
- The effect of non-independence of explanatory variables and error term and heteroskedasticity in stochastic regression models (with M. Ando), Communications in Statistics, Simulation and Computation, (2006)
- A note on bootstrapped White's test for heteroskedasticity in regression models (with M. Ando),Economics Letters (2007)
- The finite-sample performance of White's test for heteroskedasticity under stochastic regressors (with M. Ando),Communications in Statistics, Simulation and Computation, (2007)
- A simulation study of White's test for heteroskedasticity in fixed and stochastic regression models (with M. Ando), to appear in Communications in Statistics, Simulation and Computation, (2008)

$\boldsymbol{x}_i$ と ϵ_i が無相関のとき、以下の等号が成立する。

$$Cov[\boldsymbol{x}_i\boldsymbol{x}_i', \epsilon_i^2] = \kappa V[\boldsymbol{x}_i]V[\epsilon_i] \qquad \text{(Theorem (i) of Hodoshima (2004))}$$

ここで κ は、$(\boldsymbol{x}_i', \epsilon_i)'$ の任意の要素の excess kurtosis parameter,

$$\kappa = \frac{E[(Z_{ij} - E[Z_{ij}])^4]}{3(V[Z_{ij}])^2} - 1$$

Theorem

When $\boldsymbol{Z}_i \equiv (\boldsymbol{x}_i', \epsilon_i)'$ is i.i.d. and follows the class of elliptically symmetric distributions with finite fourth moments, then

$$\mathbf{H}^{-1}\mathbf{B}\mathbf{H}^{-1} - \mathbf{D} \quad \textit{is} \begin{cases} \textit{positive semidefinite} & \textit{if } \kappa > 0 \\ \textit{negative semidefinite} & \textit{if } \kappa < 0 \\ \textit{zero} & \textit{if } \kappa = 0. \end{cases}$$

ここで、$\mathbf{H}^{-1}\mathbf{B}\mathbf{H}^{-1}$ は、LSE の真の漸近共分散行列で、$\mathbf{D}$ は、$\boldsymbol{Z}_i$ が正規分布の時の漸近共分散行列である。

条件付き推測の場合：Chu (1973), "Estimation and decision for linear systems with elliptical random processes," の条件付共分散行列の証明についてのコメントと Engle, Hendry, and Richard (1983), "Exogeneity," の weak exogeneity の定義の修正 *partial weak exogeneity*

- Comments on "Estimation and decision for linear systems with elliptical random processes," IEEE Transactions on Automatic Control, (2002)
- Rejoinder to the author's reply, IEEE Transactions on Automatic Control, (2005)
- Partial weak exogeneity under nonnormality, submitted, (2006)

The joint density of the observation $\boldsymbol{x}_t$ given the past observations for $t = 1, \cdots, T$ factorizes as in

$$(1) \qquad \prod_{t=1}^{T} D(\boldsymbol{x}_t \mid \boldsymbol{X}_{t-1}, \boldsymbol{\lambda}) = \prod_{t=1}^{T} D(\boldsymbol{y}_t \mid \boldsymbol{z}_t, \boldsymbol{X}_{t-1}, \boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) \prod_{t=1}^{T} D(\boldsymbol{z}_t \mid \boldsymbol{X}_{t-1}, \boldsymbol{\lambda}_3)$$

Definition *When the conditional information matrix associated with the conditional density $D(\boldsymbol{y}_t \mid \boldsymbol{z}_t, \boldsymbol{X}_{t-1}, \boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2)$ is block-diagonal with respect to $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ and when $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_3$ are variation free in the above factorization (1), $\boldsymbol{z}_t$ is weakly exogenous for the inference of the parameters of interest $\boldsymbol{\psi}$ which are a function of $\boldsymbol{\lambda}_1$.*

The new definition of weak exogeneity is justified by the following theorem.

Theorem *The asymptotic inference of $\boldsymbol{\psi}$, the parameters of interest, based on the conditional distribution is efficient and involves no loss of information, if the conditional information matrix is block-diagonal with respect to $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ and if $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_3$ are variation-free.*